On the fundamental bandwidth limits of microwave baluns

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***Abstract*—There has been considerable progress in the construction of fundamental bandwidth limits and near optimal design realisations for several classes of passive linear electromagnetic wave devices, most notably for radar absorbent materials, antennas and meta-materials. This article seeks to place balun design on a similar footing, for an important class of baluns; in particular those which are 'perfect' and do not contain magnetically coupled transformers. For this class of baluns, the equivalent circuit is always characterised by a shunt impedance at the output of the device which is inductive in the low frequency limit. Consequently they are governed by one of the Fano [9] limits on bandwidth. The Fano integrated measure of bandwidth is proportional to the shunt inductance and is maximised when the reflection coefficient, at the balanced output port, is minimum reflection phase. Although, in theory, a minimum reflection phase high pass matching network can be shown to provide infinite fractional bandwidth in practice it is not possible to construct such a network at microwave frequencies because the shunt impedance is not purely inductive. At microwave frequencies the shunt impedance is usually realised as a shorted transmission-line section with non-negligible length. This may be approximated by a shunt inductance in parallel with a shunt capacitance over the principal operating range of the balun. This leads to a band-pass characteristic and the shunt capacitance gives rise to a second Fano bandwidth measure. It is shown that the use of both measures leads to the characteristic impedance of the transmission line, together with the balanced load impedance, determining the ultimate performance of the balun. This ultimate performance is also achieved if the reflection coefficient, looking from the balanced load into the balun, is minimum reflection phase. Suitable minimum reflection phase designs can be realised using Fano-Rhodes band-pass networks and these equivalent circuits can be compared with realistic designs. An example of such a design is presented for use over 2-18 GHz.**

***Index Terms*—microwave, baluns, filters, bandwidth**

I. Introduction

Baluns are important components of many microwave antenna systems, required in order to produce symmetric antenna patterns which remain symmetric over the operational frequency range of the antenna. As the requirement grows for antennas of larger and larger bandwidth it becomes important to seek any fundamental limits on baluns that may exist.

There has been considerable progress in establishing bandwidth performance bounds on radar absorbers [1], [2], [11], antennas [3], [4] and metamaterials [5], [6] since the seminal work of Fano [9], [10]. These types of performance bounds are characteristic of other passive structures, including certain kinds of baluns and it is the aim here to consider such baluns in this context. Munk [7] catalogued baluns, for antenna applications, according to a system employed by Nelson and Stravis [8]. The 'type 1' or 'bazooka' balun and the 'type 2' and 'type 3' balanced output baluns. This class system serves to provide a useful basis for discussion and is one which we also employ here. However, for our purposes, it is more important to distinguish between baluns which provide perfect balance over the entire frequency range, but which are usable only over a finite band due to limitations on the return loss, and those baluns which are perfectly balanced only over a limited frequency range (neighborhoods of isolated frequencies) but which present no frequency limitations on return loss. We will designate these as 'perfect baluns' and 'imperfect baluns', respectively.

Baluns may be regarded as three terminal devices, where the input terminal is a standard uni-modal waveguide port and the two output terminals support two modes between them. The output terminals may or may not be explicitly referenced to a common ground. If a well defined common ground exists then the two output modes may be designated even and odd modes with well defined characteristic impedances. If a common ground is not well defined, the return-path currents depend on external geometry and support structure and then the even mode characteristic impedance depends on this support structure. The job of the balun is to reduce the unbalanced even mode to acceptable levels. A perfect balan is one for which there is no even mode present at any frequency or, equivalently, one where each output port is phased at ± 90 degrees with respect to ground.

Imperfect baluns include the type 1 bazooka balun, the cutaway balun [7], featuring a slowly transitioning coax cable, the tapered microstrip-to-balanced strip-line and the CPW_FGP-CPS implementation of the double-Y balun [15]. These baluns are not balanced at all frequencies and become balanced only when there is a frequency dependent cancellation of the even-order mode or where the even order mode exists at a low level, small due to the relative difference in characteristic impedances of the two modes and the electrical distance in wavelengths over which this difference exists.

The type 2 and type 3 baluns are perfect baluns for which there is no even order mode at any frequency. Other examples include the Marchand balun and the slot line implementations of the double-Y balun [15]. The ideal balanced transformer, employing magnetic field coupling between the primary and secondary windings, is another example of a perfect balun. The ideal magnetically coupled transformer is a canonical device whose physical implementation at high frequency is limited by materials science rather than any requirements of causality as described by Fano. At microwave frequencies limitations are set by the finite electrical conductivity of the component metals, the unavailability of low loss high permeability materials and the sub-wavelength scales involved. Currently, upper operational frequencies are limited to a few GHz so such transformers cannot be used for many microwave applications.

As stated in [7] bandwidth limitations are set by the shunt impedance that exists for type 2 and type 3 baluns. This shunt impedance is inductive in the low frequency limit and is probably characteristic of all transformerless perfect baluns, though we are not aware of a proof of this. The two 'balanced' output terminals must be electrically symmetric under exchange of terminal position. Without a balanced transformer, all known examples of a perfect balun feature an unbroken conducting path between the two ports in order to permit balanced excitation by the input port. This unbroken path is a consequence of the requirement to 'shield' the currents excited by the input port from any even order modes that might be excited. For example, the type 2 and type 3 baluns in [7], feature an inner conductor connected to the input port which lies within a coax cable whose outer shielding forms an unbroken run between the two balanced output terminals.

Under the assumption that the shunt impedance between the balanced ports is inductive in the low frequency limit, one of the Fano bandwidth limits applies and provides a well defined bandwidth constraint. However, the shunt impedance has other generic characteristics at microwave frequencies and is better approximated by a short-circuited length of transmission line of non-zero electrical length. This leads to a band-pass model with the shunt impedance represented by a parallel inductance and capacitance. This leads in turn to a second Fano constraint which while approximate (unlike the first) serves to provide a more realistic bandwidth measure.

II. Equivalent circuit for a general transformerless perfect balun

A desirable characteristic of a balun is that it should be lossless, so that its equivalent circuit should feature a resistance only in the expression of the balun load excited by the balanced, odd mode, output. For antenna applications this is usually the radiation resistance. Although the balun is a 3-port device, the perfect balun may be regarded as a 1-port device whose input is the unbalanced port from a source of characteristic impedance Z_0, or a nearly lossless two port device whose input is a characteristic impedance Z_0 and whose output is connected to a balanced mode load R_L. For a perfect balun with no magnetically coupled transformers, the equivalent circuit is shown below. This is represented in terms of a matching network N'' connected to a reactive shunt N'. In first order representation, N' is represented by an inductor L whose output represents the balanced output of the balun into the load resistor R_L. For the present we will assume that any transmission line realisation of the inductor is electrically small so that no shunt capacitance is required in N'. The network N'' should be as close to loss-less as is practically feasible. The reflection coefficient looking from source into the load R_L is $\Gamma(\omega)$ at angular frequency ω. The backwards reflection coefficient looking from the load into the source, represented by a source load resistor Z_0, is ρ_1. The notation, for the most part, follows the conventions of Fano [9], who first presented the bandwidth theory of structures of this sort. In all that follows, a harmonic time convention is employed with time dependence $e^{j\omega t}$.

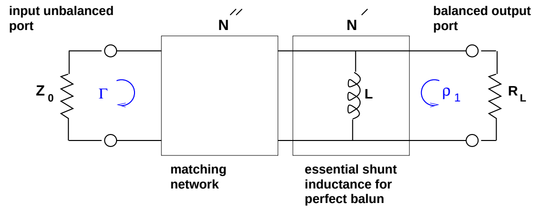

Fig. 1. First order equivalent circuit of a perfect transformerless balun

The integrated bandwidth of the balun may be defined in terms of its return loss expressed in dB weighted by the inverse square of the frequency. Network theory may be employed to produce optimal designs, with theory drawn from an early unpublished report [12] originally intended for application to radar absorbers containing frequency selective surfaces. Much of the theory is common since we are dealing with maximum bandwidth issues of passive realisable structures ([2], [3], etc.).

In order to express a measure of performance we introduce the coefficient, B_p, $0 < B_p \leq 1$ where the maximum theoretical integrated bandwidth is achieved when $B_p = 1$, the conditions for which are set out below. Let the integrated bandwidth measure, $\mathcal{I}_{bw}$, be defined by,

$$\mathcal{I}_{bw} = -\int_0^\infty \frac{\Gamma_{db}(\omega)}{\omega^2} d\omega \tag{1}$$

where $\Gamma_{db}(\omega)$ is the return loss seen at the unbalanced input port, expressed in dB as a function of angular frequency ω. Then the Bode-Fano inequality may be written as an equality,

$$\mathcal{I}_{bw} = 20log_{10}(e)B_p\frac{\pi L}{R_L} \tag{2}$$

where L is the shunt inductance described above and R_L is the load resistance and $20log_{10}(e) \approx 8.686$.

A set of necessary and sufficient conditions for $B_p = 1$ may be obtained from the original Fano theory [9] as illustrated in the appendix. In summary,

1. The network N'' should be lossless.
2. The network N'' should not feature a shunt inductance at its output terminals.
3. The reflection coefficient $\rho_1(\omega)$, looking from the load to the source, should be minimum phase, after any all-pass filter (if one exists) has been removed.

If the network is lossy then equation (2) is not valid and $\Gamma(\omega)$ must be replaced by $\rho_1(\omega)$. Conditions (1) is required in order that $|\Gamma(\omega)| = |\rho_1(\omega)|$ (in the original Fano paper [9] the analysis is conducted in terms of ρ_1). If the network N'' contains a shunt inductor of value L'' at its output terminals then the equivalent circuit is degenerate and L in (2) must be replaced by $L \to 1/(1/L + 1/L'')$. The minimum phase condition is the major theoretical requirement that this paper addresses. The requirement of removal of an all-pass filter is not strictly necessary since if an all-pass filter is placed between N' and R_L then the shunt inductance looking from load to source is increased while the magnitude of the reflection coefficient $|\rho_1(\omega)|$ remains unchanged. By definition the structure is non-minimum phase and $B_p < 1$. However, it is convenient to define L independent of any all-pass phase delay between N' and R_L since this is the assumption made when designing minimum phase filters. Hence the inclusion in condition (3).

It is very relevant that the reverse network looking from the load into the source is the same as that representing a radar absorbing material (RAM) [1], [2], [11], [12], so much of the theory developed for such applications can be applied to the balun problem. In the RAM problem the load resistor represents the impedance of free space from which a plane wave impinges on to the lossy surface. Here, the inductance L is proportional to the thickness of the absorber and to the relative permeability at zero frequency. For a radar absorber, it is often more convenient to represent N'' as lossy (rather than featuring all the loss by the single element Z_0). Since we require Γ to be replaced by ρ_1 in (2), this is not an issue.

The maximum integrated bandwidth is dependent only on the load impedance R_L and the value of L. If a transformer is available R_L can be transformed to an arbitrary value in which case there is no limit to the integrated bandwidth of the balun. Whilst balanced transformers are often unavailable, unbalanced ones can be readily fabricated (e.g. using tapered or stepped impedance transmission lines).

The integrated bandwidth $\mathcal{I}_{bw}$ is weighted heavily by the functional form of the reflection coefficient near zero frequency. This has some interesting implications. For example, if there is no matching network and the equivalent circuit is simply the inductance L in parallel with the resistance R_L, then $\mathcal{I}_{bw}$ may be easily evaluated. In this case,

$$\rho_1(\omega) = \frac{j\omega L(Z_0 - R_L) - R_L Z_0}{j\omega L(Z_0 + R_L) + R_L Z_0} \tag{3}$$

and

$$\int_0^\infty \frac{1}{\omega^2} \log_e \left(\frac{1}{|\rho_1(\omega)|} \right) = \begin{cases} \pi L/Z_0 \text{ for } Z_0 \geq R_L \\ \pi L/R_L \text{ for } Z_0 \leq R_L \end{cases} \tag{4}$$

We can replace ρ_1 by Γ and employ (2). In this case $\mathcal{I}_{bw}$ is maximised when $R_L = Z_0$ and $B_p = 1$ for $R_L \geq Z_0$ and $B_p = R_L/Z_0$ for $R_L \leq Z_0$.

In the RAM context [1],[12], which starts with the reverse network with a source impedance Z_0 that of free space, equation (2) is written differently in the form,

$$\mathcal{I}_{bw} = 20 \log_{10}(e)\, B_r \frac{\pi L}{Z_0} \tag{5}$$

In this case, $B_r = 1$ for $R_L \leq Z_0$ and $B_r = Z_0/R_L$ for $R_L \geq Z_0$. Both forms are equivalent.

Clearly, even when $B_p = 1$, most of the integral is "wasted" with a reflection coefficient that is not usefully small away from zero frequency. What is required is a network which gives a good shape to the reflection coefficient (e.g. a rectangular distribution) whilst simultaneously keeping $B_p \approx 1$ for minimum reflection phase. Suppose, for example, the reflection coefficient can be engineered to be of step form,

$$\Gamma_{db}(\omega) = \begin{cases} 0 & \text{for } \omega < \omega_0 \\ -R_{dB} & \text{for } \omega > \omega_0 \end{cases} \tag{6}$$

then,

$$\mathcal{I}_{bw} = \frac{R_{dB}}{\omega_0} \tag{7}$$

showing that an infinite bandwidth balun is theoretically possible provided that the ratio R_{dB}/ω_0 remains a constant set by (2) and provided that a high pass network is realisable. The fact it is not realisable is a consequence of other properties of the shunt impedance in N' which, above, is represented only by a single inductor.

In order to determine minimum phase requirements it is necessary to study the poles and zeros of the input impedance $Z_b(\omega)$ associated with the reflection coefficient $\rho_1(\omega)$ looking from load to source. Reversing the network representation in figure 1, we consider a ladder network representation as shown in figure 2. Here the element Z_1 represents the first shunt impedance and Z_2 the first series impedance element of the ladder. The input impedance Z_b of the balun in the reverse

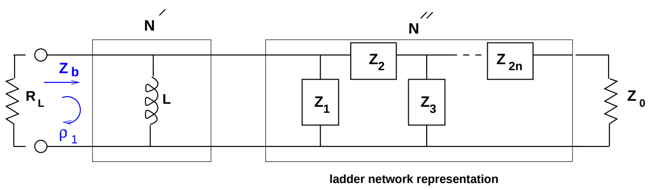

Fig. 2. Ladder network reverse representation

direction may be represented by a rational function of the Laplace transform variable, $p = j\omega$, such that

$$Z_b(p) = \frac{P(p)}{Q(p)} \tag{8}$$

for polynomial functions $P(p)$ and $Q(p)$. Clearly, Z_b must be a realisable passive function which requires that the zeros of $P(p)$ and $Q(p)$ lie in the left hand half plane. This is well described in the standard literature (e.g. [13]). However, this is not sufficient to ensure minimum phase. For minimum phase we require that the polynomial $P(p) - R_L Q(p)$ is Hurwitz stable. (Actually, this is not quite sufficient. It is also required that $P(p) - R_L Q(p)$ and $P(p) + R_L Q(p)$ share no common zeros in the right hand half plane, but since $P(p)$

and $Q(p)$ have no zeros in the right hand half plane for a passive network, this latter condition is assured for positive R_L.) The conditions for Hurwitz stability of $P(p) - R_L Q(p)$ may be found quite simply as a set of inequalities relating the equivalent circuit parameters of the network for low order networks containing inductors and capacitors. However, it is not straightforward for arbitrary networks of high order.

III. Improvements to the Model and Further Bandwidth Constraints

The integrated bandwidth can only be changed by reducing the load impedance R_L or increasing the shunt inductance L. However, methods to change L usually have an effect on the first elements, Z_1 etc. of the ladder network so this must be explicitly considered. At microwave frequencies it is essential to consider the capacitive contribution. For example if L is made large by increasing the electrical length of the balun then Z_1 becomes more capacitive at a given frequency when the frequency is small. To increase L without changing the dimensions of the inductor requires the relative permeability to be increased in which the structure is embedded since in general L is proportional to the relative permeability, assuming the relative permeability is independent of frequency. Unfortunately, this option is not available at high microwave frequencies where such magnetic materials are unavailable. The use of dispersive and artificial magnetic meta-materials is possible but the dispersion has an equivalent circuit representation which leads back to the previous formalism. Electrical metamaterials, whose relative permeability is unity at zero frequency, may permit a practical realisation of the network equivalent circuit but cannot be employed to change the above requirements.

In practice, L is realised by a structure which is better approximated by a length of short circuited transmission line with (possibly) some radiation loss. The radiation loss can be lumped in with the load resistance R_L and does not change the model, but the properties of the transmission line must be modelled at non-zero frequency. Since we are primarily interested in performance over a contiguous frequency band, the next order equivalent circuit model of the shorted transmission line may be represented as a capacitance C in parallel with L.

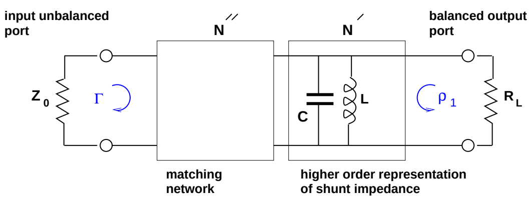

Fig. 3. Higher order equivalent circuit of a perfect transformerless balun

The existence of a non-removable capacitance gives rise to a second integrated bandwidth measure $\mathcal{J}_{bw}$,

$$\mathcal{J}_{bw} = -\int_0^\infty \Gamma_{db}(\omega) d\omega \tag{9}$$

and a second Bode-Fano inequality. Just as before, the Bode-Fano inequality may be written in the form,

$$\mathcal{J}_{bw} = 20\log_{10}(e) B'_p \frac{\pi}{CR_L} \tag{10}$$

where $0 < B'_p \leq 1$ and $B' = 1$ if and only if

1′. The network N'' is lossless.
2′. The network N'' does not feature a shunt capacitance at its output terminals. If it does, it should be lumped in with C.
3′. The reflection coefficient $\rho_1(\omega)$, looking from the balanced load to the source, is minimum phase, after any all-pass filter (if one exists) has been removed.

These are the same conditions as before with condition (2) replaced to feature degeneracy in capacitance rather than inductance. The proof of this follows the same argument as given in the appendix [10] with the logarithm of the reflection coefficient expressed as a power series in $1/p$ in the high frequency limit with coefficients A_i^∞ replacing the coefficients A_i^0 and $A_1^\infty = \sum \lambda_{0i} - \sum \lambda_{\infty i}$.

Equations (2) and (10) provide two simultaneous equations which may be solved if the reflection coefficient assumes a specific functional form. Suppose this is taken to be a rectangular distribution such that,

$$\log_e(1/|\rho_1(\omega)|) = \frac{-\Gamma_{dB}(\omega)}{20\log_{10}(e)} = \begin{cases} 0 & \omega < \omega_1 \\ h & \omega_1 \leq \omega \leq \omega_2 \\ 0 & \omega > \omega_2 \end{cases} \tag{11}$$

for lower and upper angular frequencies $\omega_1 < \omega_2$, then

$$\omega_1 = \frac{1}{2}\left(\sqrt{\left[\left(\frac{\pi}{hR_LC}\right)^2 + \frac{4}{LC}\right]} - \frac{\pi}{hR_LC}\right) \tag{12}$$

$$\omega_2 = \frac{1}{2}\left(\sqrt{\left[\left(\frac{\pi}{hR_LC}\right)^2 + \frac{4}{LC}\right]} + \frac{\pi}{hR_LC}\right)$$

If we define the fractional bandwidth,

$$\beta = \frac{\omega_2 - \omega_1}{\sqrt{\omega_2\omega_1}} \tag{13}$$

then the loss bandwidth product,

$$\beta h = \frac{\pi}{R_L}\sqrt{\frac{L}{C}} \tag{14}$$

A shorted transmission line has input impedance $Z_{in} = Z_c \tanh(pT)$. The low frequency limit may be used to define the inductance, so that

$$L = Z_c T \tag{15}$$

where Z_c is the characteristic impedance of the line and $T = d/c$ is the delay time where d is the line length and c is the propagation speed in the medium. The capacitance C may be determined from the resonant condition for which $Z_{in} \to \infty$ when $\omega = 1/\sqrt{LC}$ so that,

$$T = \frac{\pi}{2}\sqrt{LC} \tag{16}$$

in which case, in the LC pass-band approximation,

$$\sqrt{\frac{L}{C}} \approx \frac{\pi Z_c}{2} \tag{17}$$

The expression (14) is exact when the shunt impedance is represented by a parallel inductor and capacitor but only represents an approximation, with (17), for a shorted transmission line. However, it clearly shows the importance of the characteristic impedance in determining bandwidth performance. In accurate assessment of performance of a given balun, it is best only to employ the $\mathcal{I}_{bw}$ integral measure since this makes no reference to the LC approximation and makes no assumption about the functional form of the reflection coefficient.

Since impedance transformation may be accomplished over distances large compared to a wavelength using tapered transmission lines, both Z_0 and R_L can be made to take a wide range of values. If L or Z_c is fixed we would like to reduce R_L leading to a network as illustrated in figure 4. Here, the presence of a shunt capacitor is included in the Z_1 component. The auto-transformer symbol is used to show that a tapered transmission line has a common ground and is not balanced. It is not intended to imply magnetic coupling. For example,

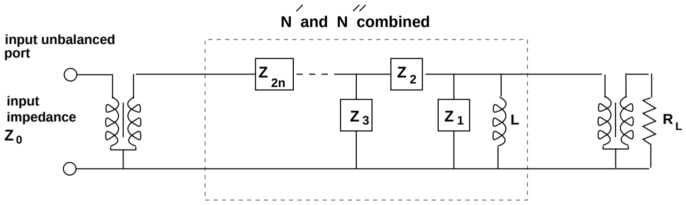

Fig. 4. Impedance transformation with a perfect balun

under this strategy for a type 2 or type 3 balun, the exterior structure which defines L or Z_c should remain fixed, but the interior part of the balun which describes the balanced output should be operated at low impedance and then transformed to the necessary higher impedance to match the antenna load. In practice the low operating impedance will be constrained by dimensional tolerances and available materials. In addition, the characteristic impedance of individual elements of the matching network may be impossible to achieve without introducing unacceptably large stray inductances and capacitances.

IV. Realisable matching networks and increasing bandwidth for fixed $\mathcal{I}_{bw}$

The other strategy to increase bandwidth is by control of the matching network without increasing either L or reducing R_L. The matching network assumes the role of defining the total required shunt capacitance. Viewed in this manner the total shunt capacitance may not be realisable if, to achieve the required inductance with a transmission line of given characteristic impedance, the line capacitance is larger than the shunt network capacitance.

As indicated in (7), at low frequencies it is theoretically possible to achieve near infinite bandwidth whilst still satisfying the causality requirement (2). To do this, the two port network comprising Z_1, Z_2, ... Z_{2n}, should ideally form a lossless high-pass filter. Minimum phase high pass filters were first designed by Fano [9], implemented as Tchebychev filters [14] and considered for radar absorber applications in [12]. In such networks the odd elements Z_1, Z_3 etc. represent inductors (in the case of Z_1, an infinite inductance) $Z_i = pL$ and the even elements Z_2, Z_4 etc. represent capacitors with $Z_i = 1/pC_i$.

For microwave applications the realisation of a perfect high pass filter is impossible, but a band pass design can be obtained by a network transformation. A number of transformations are available for this purpose where the Ls and Cs of a standard high pass filter are replaced by more complex realisable network elements. These include band-pass and transmission line element periodic forms. The requirement that any such transformation should respect the minimum phase requirement of the whole network is a feature that is not commonly considered. Fortunately, a number of theorems can be employed for this purpose. One such theorem [12] is as follows:

Theorem. If $Z_i(p)$ and $Y_i(p) = 1/Z_i(p)$ represent realisable passive impedance and admittance functions of the Laplace variable $p = j\omega$ and a two-port ladder network comprising inductors L_i and capacitors C_i terminated by R_L is minimum reflection phase, then provided

$$p'(p) \equiv \frac{U(p)}{V(p)} = \frac{Y_i(p)}{C_i} = \frac{Z_i(p)}{L_i} \text{ for all } i \tag{18}$$

then the network comprising elements $Z_i(p)$ terminated by R_L is also minimum reflection phase. Here the function $p'(p)$ is a positive function expressed by the polynomials $U(p)$ and $V(p)$ with no complex zeros in the right hand half plane. The proof relies on expressing the input impedance of the network $Z(p)$ as a rational function with polynomials expressed as a product of root factors. If,

$$Z(p'(p)) = \frac{P(p')}{Q(p')} \tag{19}$$

for polynomials $P(p'(p))$ and $Q(p'(p))$ sharing no common root factors, then $Z(p'(p))$ is minimum reflection phase if and only if $P(p'(p)) - Z_0 Q(p'(p))$ is Hurwitz stable. We have,

$$P(p'(p)) - Z_0 Q(p'(p)) = \left(\frac{1}{V(p)}\right)^N \prod_{i=1}^{N} (U(p) + p_i V(p)) \tag{20}$$

where p_i are the roots of $P(p) - Z_0 Q(p)$ prior to transformation and N is the number of such roots. When p_i are real, then p_i are positive and hence since $U(p)$ and $V(p)$ are Hurwitz stable for passive elements then so is $U(p)+p_i V(p)$. When p_i are complex then they exist as complex conjugate pairs and we consider the quadratic factors $(U(p)+p_i V(p))(U(p)+p_i^\star V(p))$ which take the form $U^2(p)+\beta U(p)V(p)+\gamma V^2(p)$ for $\beta \geq 0$, $\gamma > 0$. This polynomial is also Hurwitz stable. Because the product in (20) comprises a product of Hurwitz stable polynomials, the product is also Hurwitz stable, completing the proof.

The Richards transformation provides a representation in terms of waveguide elements [13], $p'(p) = W\tanh(pT)$ for arbitrary positive real constants W and T. Note that we explicitly introduce the scaling factor W with units of frequency to ensure that p' and p have the same units. This transformation replaces inductors by sections of shorted transmission line and capacitors by sections of open-ended transmission line.

Since an open ended waveguide has input impedance $Z_{in} = Z_0^{(c)}/\tanh(pT)$ and a closed ended waveguide has input impedance $Z_{in} = Z_0^{(c)}\tanh(pT)$ for characteristic impedance $Z_0^{(c)}$ it follows that each transmission line element has a characteristic impedance $Z_i^{(c)}$ given by $Z_i^{(c)} = 1/(WC_i)$ for the open ended sections and $Z_i^{(c)} = WL_i$ for the shorted sections. Each element must be of equal electrical length such that if β represents the wavenumber at frequency ω in a guide of length l, then $\beta l = \omega T$. The constant T represents the (1-way) propagation time and may be chosen arbitrarily to scale the frequency range of the balun but it has no effect on bandwidth. The scaling constant W (with units of frequency) may be set so that the impedance of the shorted transmission line and parallel LC combination match as $\omega \to 0$ and such that the position of the first pole matches. In this case, $W = 1/T$ and the reflection coefficient becomes a periodic function of frequency with an infinite number of pass bands. The Richards elements can in principal be implemented at microwave frequencies, but the formalism ignores stray capacitances and inductances which are usually sufficiently large that an LC band-pass approximation is equally as accurate.

Without the use of magnetic materials with relative permeability $\mu_r \gg 1$ it is difficult to achieve characteristic impedances in a TEM line much greater than that of free space, $Z_0^{(fs)} \approx 377$ ohms, at microwave frequencies. Methods to obtain high characteristic impedances include the use of conductor strip widths or wire diameters less than a tiny fraction of a millimetre, or the use of convoluted inductor-like structures but in both cases the achievable characteristic impedance is limited, especially if ohmic losses are small.

V. The Fano-Rhodes band pass matching network

Fano [10] considered a maximum bandwidth ladder network for transfer of energy from a resistive source to a load consisting of a capacitor and resistor in parallel. This design meets the minimum phase requirement with a reflection coefficient that is the ratio of two Hurwitz stable polynomials. Although this is a low pass structure, and does not have the required shunt inductive behaviour, it can be transformed to one that does using appropriate filter transforms that maintain minimum phase.

As far as we are aware, Fano was first to describe the design of a Tchebychev ladder network meeting this minimum phase requirement. More recently, explicit network parameters have been derived for an arbitrary n_{th} order filter of this sort, for example as described in Rhodes [14]. Rhodes also considers a number of other filters, based on normalised low-pass prototypes, which exhibit the required Hurwitz stability in the numerator polynomial of the impedance function. These include the *equidistant linear phase* polynomial, which includes the Tchebychev polynomials as a special case, and *arbitrary phase* polynomials, both of which may have application here. However, these latter types are rather more complicated and no explicit network parameter formulas are currently available. For our purposes, all such singly loaded loss-less ladder networks whose reflection coefficient is a rational function of the ratio of two Hurwitz polynomials will be termed *Fano-Rhodes* ladder networks.

Using standard filter transforms, it is possible to convert a low pass filter into a dual form and transform either the original or its dual into a high pass or band pass filter. A property of the transformation is that the bandwidth and shape of its frequency characteristics is conserved (under suitable definitions) and hence a structure that has optimal bandwidth in its original form has an optimal bandwidth in its transformed form. The basic transformations employed in filter theory are usually designed to include at most one capacitor and one inductor to each element Z_i. This is not necessary, but we will not seek to generalise here.

The matching network scheme hinges on making the first shunt element of the (transformed) Fano-Rhodes network equal to the transmission line shunt impedance, comprising L and any more realistic network representation of the 'exterior' conducting path of the balun that gives rise to L. In its simplest form this is the inductor L and a parallel capacitance C, suitable for a pass-band design. In what follows we take the characteristic source impedance of the reverse network, $Z_0^{(in)} = R_L$ and the termination resistance $R = Z_0$ without assuming $R_L = Z_0$. In this case Γ should be replaced by ρ_1 in (2). Figure 5 shows the nature of the ladder network required to compensate for the shunt impedance assuming a high-pass (shunt inductor) or band-pass (shunt inductor and capacitor) Fano-Rhodes network. Z_{in} is the input impedance, as illustrated.

Rhodes [14] analysis employs a description of the network parameters in terms of series inductors and perfect inverters for the loss-pass primitive forms. This raises a question on the realisability of inverters. There are two issues here. Firstly, although a single inverter cannot be perfectly realised using a finite number of inductors and capacitors, there are standard approximations which are valid over a limited frequency band (e.g. see chapter 15 of [16]). If the bandwidth of the inverter approximation is larger than the bandwidth of the balun these methods can be employed. Secondly, it is relatively straight forward to show that, subject to certain limitations that are of no consequence here, a ladder network containing inverters can be realised using inductors and capacitors with an explicit relationship between the two equivalent network representations.

The Fano-Rhodes Tchebychev (FRT) network is a general n_{th} order filter capable of meeting the optimal matching requirements set out by Fano [10] where explicit formulae

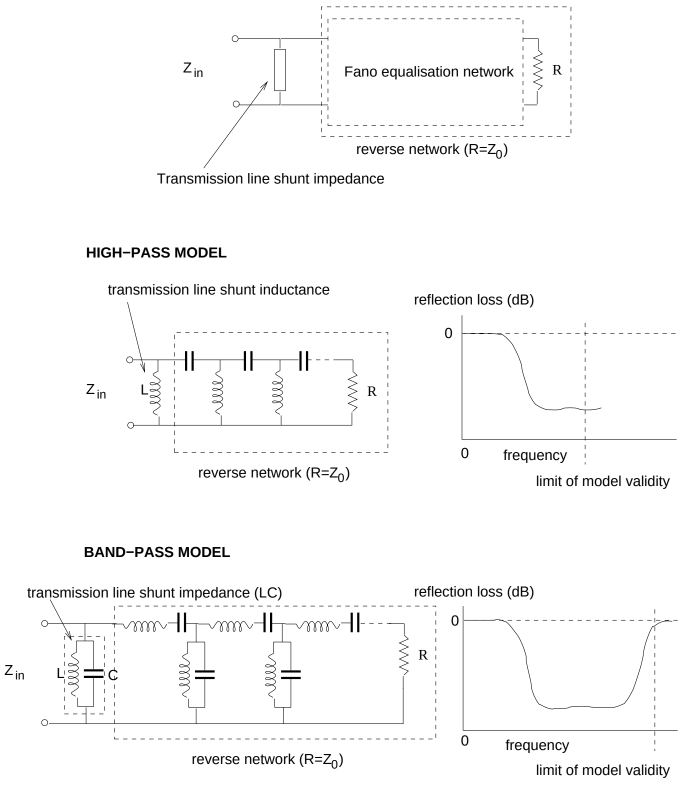

Fig. 5. Fano networks appropriate to balun design.

are available for the component values. This is referred to as the *minimum phase* S_{11} *Tchebychev prototype* in [14]. The simpler *minimum phase* S_{11} *Maximally Flat prototype* may be regarded as a special case. After suitable transformation the FRT can be used as a prototype for the construction of a matching network for the balun. To make our use of the theory clear, we will consider the argument in various stages, following the sequence of forms (a)-(c) illustrated in figure 6, below.

Stage (a)

We first assume a canonical form in terms of series inductors and inverters, as illustrated in the inverter form of the original low-pass design, figure 6-a. Note that an inverter of characteristic K is defined as a device for transforming between two impedances, Z and Z' under the transformation,

$$Z = \frac{K^2}{Z'} \tag{21}$$

The following material comes from [14] (section 2.7), modified to include a non-unity source impedance Z_0 and non-unity cut-off frequency ω_0 (following the scaling rules, section 2.12). For a Tchebychev filter of order n, the transmission coefficient S_{12} from the source impedance Z_0 into the load impedance R' is assumed to be of the form,

$$|S_{12}(j\omega)|^2 = \frac{A}{1+\epsilon^2 T_n^2(\omega/\omega_0)} \tag{22}$$

where T_n is a Tchebychev polynomial of order n in the real frequency variable ω. The coefficients A and ϵ are real numbers; ϵ may be of any value describing the ripple level and out-of-band tail-off, $A \leq 1$ describing the filter depth. In this form, we explicitly include the cut-off frequency ω_0 (in [14], ω is assumed dimensionless and frequency scaling is performed after the formulation).

The subsidiary parameters η and ξ are then defined by,

$$\eta = \sinh\left(\frac{1}{n}\sinh^{-1}\frac{1}{\epsilon}\right) \tag{23}$$

$$\xi = \sinh\left(\frac{1}{n}\sinh^{-1}\sqrt{\frac{1-A}{\epsilon}}\right) \tag{24}$$

where $\eta > \xi > 0$. When $S_{11}(p)$ is minimum phase it takes the form,

$$S_{11}(p) = \prod_{r=1}^{n}\left(\frac{p/\omega_0 + j\cos[\sin^{-1}(j\xi)+(2r-1)\pi/2n]}{p/\omega_0 + j\cos[\sin^{-1}(j\eta)+(2r-1)\pi/2n]}\right) \tag{25}$$

The load impedance may be shown [14] to be,

$$R'(\eta,\xi) = Z_0^{(in)}\frac{\eta+\xi}{\eta-\xi} \tag{26}$$

If the other network parameters are the series inductors L'_r (for $1 \leq r \leq n$) and inverter parameters $K_{r,r+1}$ (for $1 \leq r \leq n-1$) then [14],

$$L'_r = \frac{2Z_0^{(in)}\sin[(2r-1)\pi/(2n)]}{\omega_0(\eta-\xi)} \qquad \text{for } 1 \leq r \leq n. \tag{27}$$

and

$$K_{r,r+1} = \frac{Z_0^{(in)}\sqrt{\xi^2+\eta^2-2\eta\xi\cos(r\pi/n)+\sin^2(r\pi/n)}}{\eta-\xi} \tag{28}$$
$$\text{for } 1 \leq r \leq n-1$$

The input impedance for the original low-pass inverter form may be represented by the continued fraction,

$$Z_{in}^{(a)} = pL'_1 + \cfrac{K_{12}^2}{pL'_2 + \cfrac{K_{23}^2}{pL'_3 + \ldots + \cfrac{K_{n-2,n-1}^2}{pL'_{n-1} + \cfrac{K_{n-1,n}^2}{pL'_n + R'}}}} \tag{29}$$

It may also be represented, for the LC form represented in figure 6-a, as the continued fraction

$$Z_{in}^{(a)} = pL_1^{(a)} + \cfrac{1}{pC_2^{(a)} + \cfrac{1}{pL_3^{(a)} + \ldots + \cfrac{1}{pC_{n-1}^{(a)} + \cfrac{1}{pL_n^{(a)} + R^{(a)}}}}} \tag{30}$$

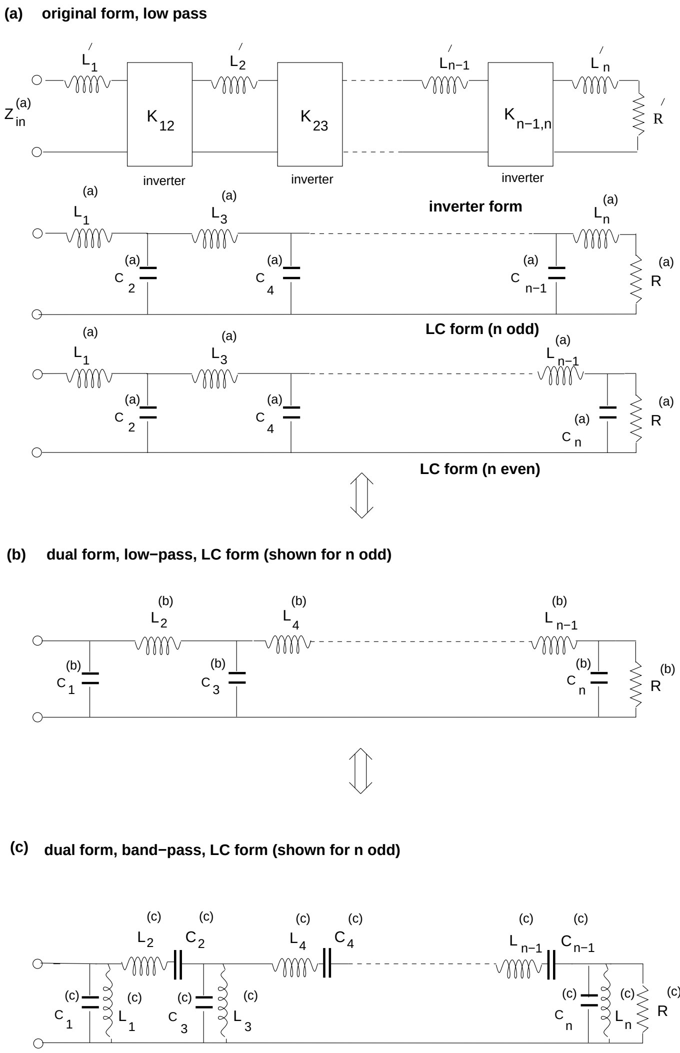

Fig. 6. The dual low-pass and band-pass forms of the FRT network

Comparison of these two forms shows that, for n odd,

$$\begin{aligned}
L_1^{(a)} &= L_1' \\
C_2^{(a)} &= \frac{L_2'}{K_{12}^2} \\
L_3^{(a)} &= \frac{K_{12}^2 L_3'}{K_{23}^2} \\
&\cdot \\
&\cdot \\
C_{n-1}^{(a)} &= \frac{L_{n-1}'}{K_{n-2,n-1}^2}\left(\frac{L_{n-2}'}{L_{n-2}^{(a)}}\right) \\
&= \frac{K_{n-3,n-2}^2 \ldots K_{23}^2}{K_{n-2,n-1}^2 K_{n-4,n-3}^2 \ldots K_{12}^2} L_{n-1}' \\
L_n^{(a)} &= \frac{L_n'}{K_{n-1,n}^2}\left(\frac{L_{n-1}'}{C_{n-1}^{(a)}}\right) \\
&= \frac{K_{n-2,n-1}^2 K_{n-4,n-3}^2 \ldots K_{12}^2}{K_{n-1,n}^2 K_{n-3,n-2}^2 \ldots K_{23}^2} L_n' \\
\text{and} \quad R^{(a)} &= \frac{R'}{K_{n-1,n}^2}\left(\frac{L_{n-1}'}{C_{n-1}^{(a)}}\right) \\
&= \frac{K_{n-2,n-1}^2 K_{n-4,n-3}^2 \ldots K_{12}^2}{K_{n-1,n}^2 K_{n-3,n-2}^2 \ldots K_{23}^2} R'
\end{aligned} \tag{31}$$

where the number of K^2 terms is the same in numerator and denominator for the inductors $L_r^{(a)}$ $(r = 1, 3, 5, ..., n)$ but one less in the numerator for the capacitors $C_r^{(a)}$ $(r = 2, 4, ..., n-1)$.

When n is even,

$$\begin{aligned}
L_1^{(a)} &= L_1' \\
C_2^{(a)} &= \frac{L_2'}{K_{12}^2} \\
L_3^{(a)} &= \frac{K_{12}^2 L_3'}{K_{23}^2} \\
C_4^{(a)} &= \frac{K_{23}^2 L_4'}{K_{12}^2 K_{34}^2} \\
&\cdot \\
&\cdot \\
L_{n-1}^{(a)} &= \frac{K_{n-3,n-2}^2 K_{n-5,n-4}^2 \ldots K_{12}^2}{K_{n-2,n-1}^2 K_{n-4,n-3}^2 \ldots K_{23}^2} L_{n-1}' \\
C_n^{(a)} &= \frac{K_{n-2,n-1}^2 \ldots K_{23}^2}{K_{n-1,n}^2 K_{n-3,n-2}^2 \ldots K_{12}^2} L_n' \\
\text{and} \quad \frac{1}{R^{(a)}} &= \frac{K_{n-2,n-1}^2 \ldots K_{23}^2}{K_{n-1,n}^2 K_{n-3,n-2}^2 \ldots K_{12}^2} R'
\end{aligned} \tag{32}$$

Stage (b)

We now consider the dual form. We employ a general theorem of reciprocal networks, as given by Bartlett [17]. This shows that given any ladder network composed of series impedance elements S_i and shunt impedance elements T_i, the dual network composed of reciprocal shunt elements (reciprocal with respect to the impedance k), k^2/S_i, and reciprocal series elements k^2/T_i has an input impedance which is itself reciprocal to the original. Referring to figure 7, $Z_a = k^2/Z_b$. For

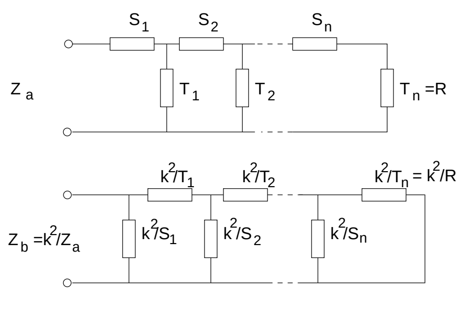

Fig. 7. The relationship between dual forms of a ladder network (Bartlett's theorem)

our purposes, we assume the series elements with impedance S_i are inductors and the shunt elements with impedance T_i

are capacitors. Referring to figure 6-b, we therefore define,

$$\begin{array}{rcl} C_1^{(b)} &=& L_1^{(a)}/k^2 \\ L_2^{(b)} &=& k^2 C_2^{(a)} \\ C_3^{(b)} &=& L_3^{(a)}/k^2 \\ && . \\ && . \\ L_{n-1}^{(b)} &=& k^2 C_{n-1}^{(a)} \\ C_n^{(b)} &=& L_n^{(a)}/k^2 \\ R^{(b)} &=& k^2/R^{(a)} \end{array} \quad (33)\ (34)$$

for some as yet unspecified real impedance k. The input impedance is then given by,

$$Z_{in}^{(b)} = \frac{k^2}{Z_{in}^{(a)}} \quad (35)$$

and the reflection coefficient of the dual form is given by,

$$S_{11}^{(b)} = \frac{k^2 - Z_0^{(in)} Z_{in}^{(a)}}{k^2 + Z_0^{(in)} Z_{in}^{(a)}} \quad (36)$$

We now express an important lemma. Writing $Z_{in}^{(a)} \equiv U/V$ as a rational function with Hurwitz stable polynomials $U(p)$ and $V(p)$ in the Laplace transform variable $p = j\omega$, we may rewrite $S_{11}^{(b)}$ as,

$$S_{11}^{(b)} = -\frac{U - (k^2/Z_0^{(in)})V}{U + (k^2/Z_0^{(in)})V} \quad (37)$$

Since the reflection coefficient for the original network, $S_{11}^{(a)} = (U - Z_0^{(in)}V)/(U + Z_0^{(in)}V)$ is taken to be of minimum phase (Hurwitz U/V form), $S_{11}^{(b)}$ is of minimum phase if $k \le Z_0^{(in)}$. This follows from a stability theorem that if $U(p)$ and $V(p)$ are stable and if $U(p) - Z_0^{(in)}V(p)$ is stable then so is $U(p) - \alpha V(p)$ for $0 \le \alpha \le Z_0^{(in)}$.

Stage (c)

The final transformation makes use of another standard network transformation, used to transpose a low pass to a band pass filter. The transformation in the Laplace transform variable (see, for example, section 2.12 of [14][1]) is,

$$p \to \alpha \left(\frac{p}{\omega_c} + \frac{\omega_c}{p} \right) \quad (38)$$

where

$$\begin{array}{rcl} \omega_c &=& \sqrt{\omega_1 \omega_2} \\ \alpha &=& \dfrac{\omega_0 \sqrt{\omega_1 \omega_2}}{\omega_2 - \omega_1} \end{array} \quad (39)$$

where ω_1 and ω_2 are, respectively, the lower and upper cut-off frequencies to the pass band and ω_0 is the cut-off frequency of the original low-pass filter. Under this transformation, every L in the original network becomes a series L and C, and every C in the original network becomes a parallel shunt L and C. Referring to figure 6-c, the new component values are given by,

$$\left.\begin{array}{l} L_r^{(c)} = \dfrac{\alpha L_r^{(b)}}{\omega_c} \\ C_r^{(c)} = \dfrac{1}{\alpha L_r^{(b)} \omega_c} \end{array}\right\} \quad \text{for } r \text{ even. Series components.}$$

$$\left.\begin{array}{l} L_r^{(c)} = \dfrac{1}{\alpha C_r^{(b)} \omega_c} \\ C_r^{(c)} = \dfrac{\alpha C_r^{(b)}}{\omega_c} \end{array}\right\} \quad \text{for } r \text{ odd. Shunt components.}$$

$$R^{(c)} = R^{(b)} \quad (40)$$

VI. FRT BANDPASS EXAMPLE, $N = 3$

Here we consider the case $N = 3$ which is of order suitable for balun construction. The three impedance elements Z_1, Z_2 and Z_3 form the matching network. In this case, the auxiliary parameters η and ξ are, from (23) and (24),

$$\eta = \sinh\left(\frac{1}{3}\sinh^{-1}\frac{1}{\epsilon}\right) \quad (41)$$

$$\xi = \sinh\left(\frac{1}{3}\sinh^{-1}\sqrt{\frac{1-A}{\epsilon}}\right) \quad (42)$$

$$K_{12} = \frac{Z_0^{(in)}\sqrt{\xi^2 + \eta^2 - \xi\eta + 3/4}}{\eta - \xi} \quad (43)$$

$$K_{23} = \frac{Z_0^{(in)}\sqrt{\xi^2 + \eta^2 + \xi\eta + 3/4}}{\eta - \xi} \quad (44)$$

Proceeding to stage (c), the component values are given by,

$$\begin{array}{l} C_1^{(c)} = \dfrac{Z_0}{(\eta - \xi)(\omega_2 - \omega_1)k^2} \\ L_1^{(c)} = \dfrac{(\eta - \xi)(\omega_2 - \omega_1)k^2}{\omega_1\omega_2 Z_0} \\ C_2^{(c)} = \dfrac{(\eta - \xi)(\omega_2 - \omega_1)K_{12}^2}{2\omega_1\omega_2 Z_0 k^2} \\ L_2^{(c)} = \dfrac{2Z_0 k^2}{(\eta - \xi)(\omega_2 - \omega_1)K_{12}^2} \\ C_3^{(c)} = \dfrac{Z_0 K_{12}^2}{(\eta - \xi)(\omega_2 - \omega_1)k^2 K_{23}^2} \\ L_3^{(c)} = \dfrac{(\eta - \xi)(\omega_2 - \omega_1)k^2 K_{23}^2}{\omega_1\omega_2 K_{12}^2 Z_0} \\ R^{(c)} = \dfrac{k^2 K_{23}^2(\eta - \xi)}{Z_0 K_{12}^2(\eta + \xi)} \end{array} \quad (45)$$

[1]Note that Rhodes assumes unit cut-off frequency for the original low-pass filter.

By way of example, suppose we assume $f_1 = \omega_1/2\pi = 3.0$ GHz, $f_2 = \omega_2/2\pi = 20.0$ GHz, $k = Z_0^{in} = 60$ ohms, $A = 0.9$ and take values of $\epsilon = 0.01, 0.04, 0.16$ and 0.64. Note that Z_0^{in} represents the characteristic impedance of the balanced output of the balun. Table I shows the equivalent circuit parameters and numerically estimated (from the reflection coefficient) values of B_p integrated over the range zero to 50 GHz. Note that R represents the required characteristic impedance of the unbalanced input to the balun.

The reflection coefficient, as a function of frequency, is plotted in figure 8. As we expect, the performance improves as the value of $L_1^{(c)}$ increases, which occurs as the parameter ϵ is made smaller. In design of a balun, the inductor $L = L_1^{(c)}$ shunting the balanced output load $Z_0^{(in)} = R_L$ is likely to be a given so the remaining parameters should be adjusted to match.

ϵ	equivalent circuit values							B_p
	$C_1^{(c)}$ /pF	$L_1^{(c)}$ /nH	$C_2^{(c)}$ /pF	$L_2^{(c)}$ /nH	$C_3^{(c)}$ /pF	$L_3^{(c)}$ /nH	$R^{(c)}$ /Ω	
0.01	0.0718	5.884	1.272	0.332	0.0475	8.895	56.32	1.000
0.04	0.1217	3.468	0.914	0.462	0.0831	5.081	52.86	1.000
0.16	0.2217	1.904	0.789	0.535	0.169	2.501	46.53	1.000
0.64	0.5338	0.791	1.144	0.369	0.476	0.887	35.77	1.000

Table I
EQUIVALENT CIRCUIT VALUES FOR BAND-PASS MINIMUM PHASE EXAMPLE

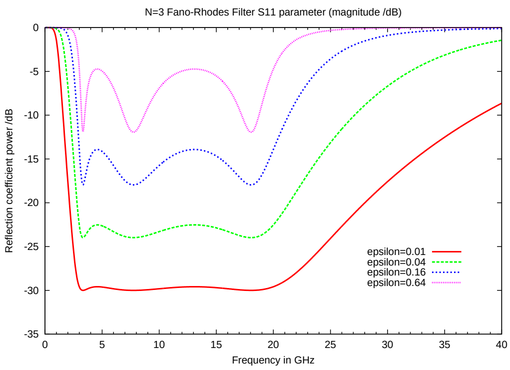

Fig. 8. Reflection coefficient under variation of ϵ in the $n = 3$ transformed Fano-Rhodes Tchebychev filter (0-40 GHz).

VII. A PRACTICAL BALUN EXAMPLE

Although the pass-band model can be realised with discrete components at low frequencies it can serve only as a design aid at microwave frequencies where the demands of practical implementation make the model almost impossible to achieve. In this section, CST [18] is used to design a microstrip to modified slot-line Marchand balan corresponding approximately to an $N = 3$ band pass ladder network. This is not intended to achieve $B_p = 1$ (which may be impossible in practice) but is nevertheless intended to show good performance. The design is intended to provide operation between 2.0 GHz and 18.0 GHz which is a respectable bandwidth for a Marchand-style balun. The $N = 3$ design includes an additional shunt component over the standard Marchand form. The characteristic impedance of the unbalanced microstrip is approximately 50 ohms whilst that of the modified slotline is approximately 60 ohms. The use of a floating strip on the top side of the substrate is employed to achieve this without the width of the slotline being unacceptably small. The slotline is 0.2 mm wide with 0.8 mm wide microstrip. The substrate is taken to have relative permittivity $\epsilon_r = 3.38$, assumed lossless. Substrate thickness is 0.35 mm. All conducting track is assumed to be 0.017 mm thick perfect conductor.

Referring to figure 2, the $N = 3$ pass-band model assumes an inductor $L = L_1$ and three impedance elements Z_1, Z_2 and Z_3 where $Z_1 = 1/j\omega C_1$ is purely capacitive, Z_2 represents the open-circuit stub by a series inductor and capacitor $Z_2 = j\omega L_2 + 1/j\omega C_2$ and Z_3 a parallel inductor and capacitor $1/Z_3 = j\omega C_3 + 1/j\omega L_3$. Z_3 is represented by a thin conductor of width 0.15 mm and length 5.1 mm with the ground plane beneath it removed. The end of the line is connected by a conducting via to the ground plane beneath. This high impedance line is similar to co-planar waveguide without ground plane. L_1 is defined by the cavity comprising a conducting shield and gap in the ground plane at the end of the stripline. In construction, the shields should be bolted together through the substrate, ensuring good electrical contact with the ground plane. The shield serves to reduce radiation losses and well define the characteristics of the balun when the circuit board is in proximity to other structure. C_1 is generated as a result of the gap-plus-screen and the stray capacitances near the feeding elements. The design is shown in figures 9 to 11, where the more important dimensions are illustrated. Figures 12 and 13 show the CST predicted return and transmission loss. Note that all losses are radiative since the model assumes a lossless dielectric and lossless (perfect) conductors.

The shunt inductance L_1 may be computed by examination of the low frequency phase response of the reflection coefficient of the screened gap region. The screened gap behaves as a waveguide below cutoff so its inductance is controlled more by the screen dimensions than the length of the gap. A separate CST analysis is required, with the analysed structure shown in figure 14. Note the position of the port; the inductance is defined at the point where the microstrip excites the slotline, just in front of the screen. In this model the microstrip tracks and stubs have been removed, leaving only the modified slotline excitation of the cavity. The magnitude of the reflection coefficient is close to unity, with predicted reflection phase shown in figure 15. The inductance, L, is given by,

$$\frac{d\Phi}{d\omega} = -\frac{2L}{Z_0} \text{ as } \omega \to 0 \tag{46}$$

where $Z_0 = 60$ Ohms. The results indicate a phase gradient of -8.2×10^{-8} degrees/Hz which implies $L_1 \approx 5.6$ nH. Numerical estimation of $\mathcal{I}_{bw}$ between 0.5 GHz (below which the reflection coefficient is negligibly small) and 20 GHz (above which no data for this model is available) indicates $B_p \approx 0.50$, assuming that the reflection coefficient is small above 20 GHz. The capacitance C_1 can be estimated by the frequency ω_z at which the reflection phase is zero at which point $\omega_z = 1/\sqrt{L_1 C_1}$. This occurs at approximately 7.6 GHz, leading to a value of $C_1 \approx 0.078$ pF and $Z_c \approx 170$ Ohms using (17).

It is instructive to view the kind of values predicted by the FRT equivalent circuit model shown previously, since the performance of the real balun design is not as good. Using (17) the tabulated values of L_1 and C_1 define a characteristic impedance $Z_c \approx 180$ Ohms, probably close enough to the above value not to be an issue. The tabulated L_2 and C_2 define an open circuit transmission line with characteristic impedance $Z_c \approx 10$ Ohms and the tabulated L_3 and C_3 define a shorted line with characteristic impedance $Z_c \approx 270$ Ohms. The electrical length is constant for all elements Z_1, Z_2 and Z_3 with estimated $d = 9.7$ mm. On the other hand the real balun example features an open circuit stub approximated by 2.7 mm wide microstrip which has a characteristic impedance of about 20 ohms. Similarly, the via-terminated 0.15 mm thin line has a characteristic impedance of about 170 Ohms (evaluated by CST, but very similar to coplanar waveguide with no ground plane). Unfortunately, respectively lowering and raising these characteristic impedances is not feasible on this substrate material.

VIII. Conclusions

The conditions for construction of a bandwidth-optimal perfect balan are given in terms of a minimum reflection phase band pass filter equivalent circuit. Other minimum reflection phase networks are also possible, but have not been considered here. The representation is applicable to any perfect balan for which the balanced output port is inductive in the low frequency limit. In reality, for microwave ultra wide band applications, it is likely that $B_p = 1$ designs are not possible, but this sets the "gold standard" for reference purposes. Practical realisations have reduced performance due to a number of reasons, but chiefly due to the difficulty of achieving elements with both sufficiently high and (to a lesser extent) sufficiently low characteristic impedances and the difficulty of connecting them together without introducing unwanted parasitics. A practical 2-18 GHz balun is simulated using CST, for comparison purposes, with an estimated value of $B_p = 0.5$.

In addition to the requirement to come as close as possible to achieving minimum reflection phase (looking into the balanced port towards the unbalanced port through an all-pass filter), it is also advantageous to employ a balanced output port whose characteristic impedance is as low as possible and transformed using an impedance transformer to the required load impedance. This will also be set by practical constraints, e.g. track widths and mechanical tolerances. Finally we would remark that for this application, as for many others, there exists the general desire to obtain lossless high relative permeability materials at microwave frequencies. These could be used to realise large inductances without large capacitance, or equivalently to achieve high characteristic impedance transmission lines. Unfortunately, this still remains something of a 'holy grail'.

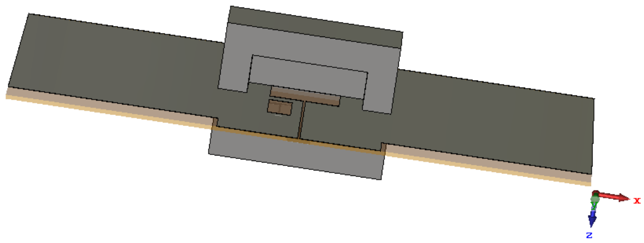

Fig. 9. Perspective view (grey represents conductor over substrate)

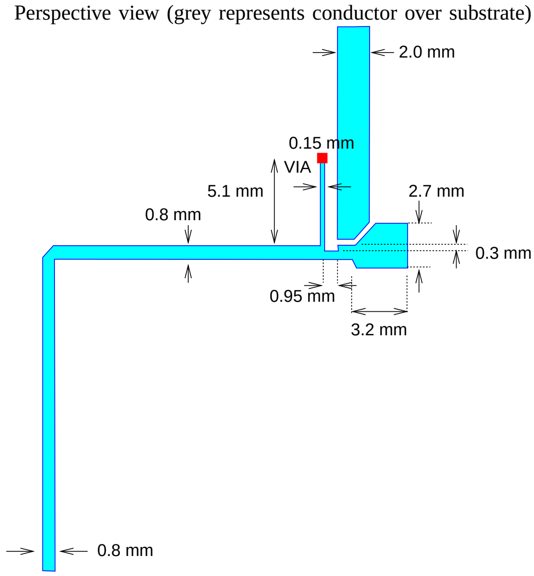

Fig. 10. Top plan view (blue represents conductor over substrate)

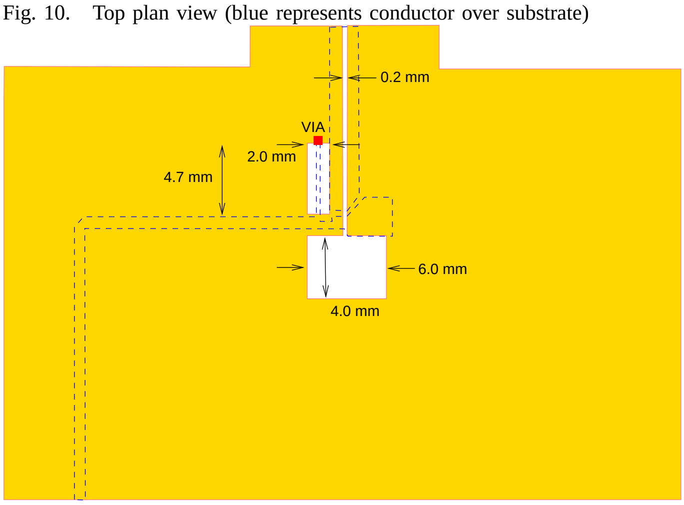

Fig. 11. Plan view showing regions of conductor removed from ground plane

IX. Appendix 1. The requirements for optimal integrated bandwidth

Previously three necessary and sufficient conditions were specified in order to ensure a maximum integrated bandwidth defined by $B_p = 1$. The Bode-Fano bandwidth inequalities

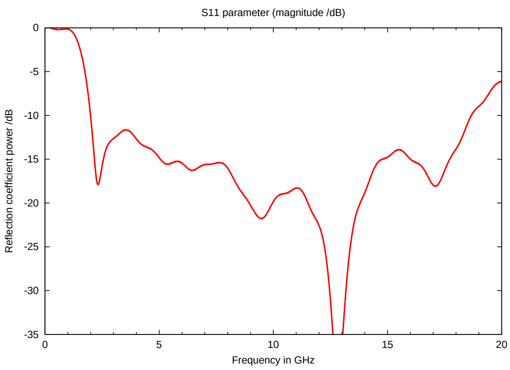

Fig. 12. Predicted return loss, S11 (0-20 GHz)

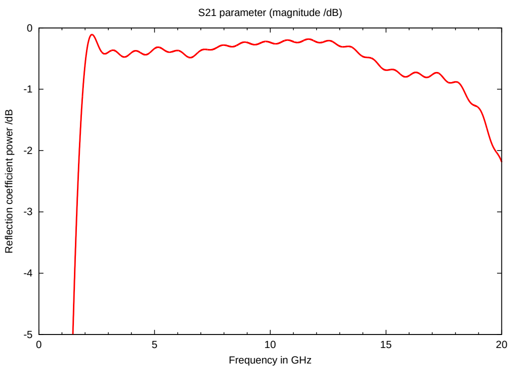

Fig. 13. Predicted transmission loss, S21 (0-20 GHz)

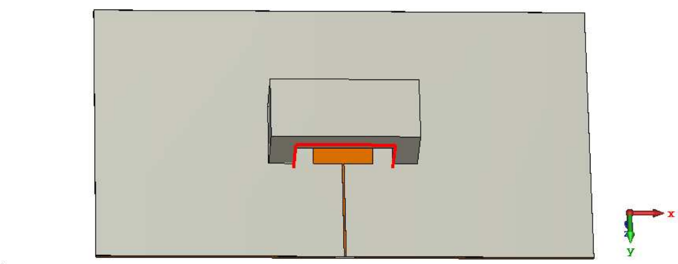

Fig. 14. CST model for analysis of the shunt element (screened gap). Excitation port shown by red rectangle about 0.5 mm from screen.

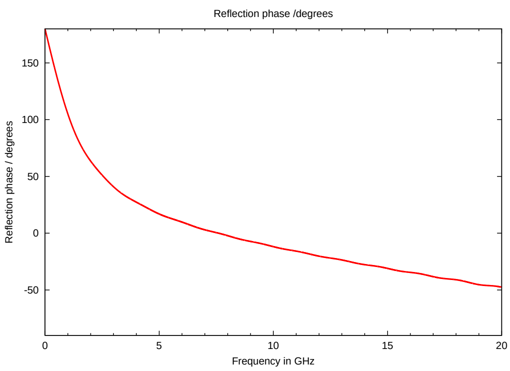

Fig. 15. Predicted reflection phase of the L_1 component (0-20 GHz)

are well known, but not so the conditions for equality which come from Fano's original analysis. Referring to figure 1 and to [9], the logarithm of the inverse of the backward reflection coefficient ρ_1 may be written as,

$$\log_e(1/\rho_1) = -1 + A_1^0 p + A_3^0 p^3 + \dots \tag{47}$$

written as a Taylor expansion near zero frequency, where $p = j\omega$. Of importance is the coefficient A_0^1 which is independent of the network N'' assuming that N'' does not have a shunt inductor at its output port (this is an example of degeneracy which Fano considers in greater generality than required here). If it does, then it should be lumped in with L. If there is no such degeneracy,

$$A_1^0 = \sum_i p_{0i}^{-1} - \sum_i p_{\infty i}^{-1} \tag{48}$$

where p_{0i} are the zeros of ρ_1 and $p_{\infty i}$ are the poles of ρ_1 which are independent of the network N''. Fano shows that,

$$\int_0^\infty \frac{1}{\omega^2} \log_e\left(\frac{1}{|\rho_1(\omega)|}\right) d\omega = \frac{\pi}{2} F_1^0 = \frac{\pi}{2}\left(A_1^0 - 2\sum_i p_{ri}^{-1}\right) \tag{49}$$

where p_{ri} are the real parts of any zeros of ρ_1 in the right hand half plane. The integral is positive and A_1^0 depends only on the network N' inductor so the sum over p_{ri} always reduces the value of the integral. Furthermore, each p_{ri} is positive so we require there to be no zeros in the right hand half plane. This is a statement of the minimum phase definition of the reflection coefficient ρ_1.

Consequently, the integral attains its maximum value provided N'' contains no shunt inductance at its output and provided ρ_1 is minimum phase. In this case the integral takes the value $\pi A_1^0/2$ which is dependent only on the value of L which is the only element of N'. In the limit that $\omega \to 0$, only the inductor L contributes to the reflection coefficient ρ_1,

$$\rho_1 \to \frac{pL - R_L}{pL + R_L} \tag{50}$$

which has one pole and one zero, $p_{\infty 1} = -R_L/L$ and $p_{01} = R_L/L$ so that $A_1^0 = 2L/R_L$. Thus we obtain,

$$\int_0^\infty \frac{1}{\omega^2} \log_e\left(\frac{1}{|\rho_1(\omega)|}\right) d\omega = \frac{\pi L}{R_L} \tag{51}$$

If the network N'' is lossless then $|\Gamma| = |\rho_1|$ and we obtain equality in the Bode-Fano result.

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