The Theory and Design of Provably Optimal Bandwidth Radar Absorbent Materials (RAM) Using Dispersive Structures and/or Frequency Selective Surfaces (FSS)

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The Theory and Design of Provably Optimal Bandwidth Radar Absorbent Materials (RAM) using Dispersive Structures and/or Frequency Selective Surfaces (FSS)

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Abstract

Performance coefficients, design tools and analysis is provided for the synthesis of optimal radar absorbent materials (RAM). The importance of equivalent circuits is emphasised. The basic theory is quite general and valid for metamaterial and frequency selective surfaces (FSS) composites containing piece-wise isotropic layered materials.

1 Introduction

In this article we assume a radar absorbent material (RAM) is one which is designed for plane wave reflection, is flat (i.e. with a surface which has a radius of curvature large compared with the largest wavelength of operation), is terminated by a perfect conductor and is composed of passive linear constituents. Such a RAM may be said to be optimal if it achieves the greatest possible absorption over the largest possible bandwidth for a given composite RAM thickness. More precisely (e.g. [1]), an optimal RAM is required to be *minimum reflection phase.* This is a general necessary condition. More specific optimality requirements may be attached, for example if the absorption characteristic is required to be as near as possible to a rectangular distribution.

Rozanov [1] provided the minimum reflection phase condition in terms of a bandwidth integral, defined over the free-space wavelength,

$$\int_0^\infty -\Gamma_{db}(\lambda)\, d\lambda \leq 40\pi^2(log_{10}e)\mu_e d \qquad (1)$$

where $\Gamma_{db}(\lambda)$ is the reflection coefficient in dB at normal incidence as a function of the free space wavelength λ, d is the composite RAM thickness and μ_e is the

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effective relative magnetic permeability at zero frequency. This is a real quantity. If the RAM is composed of multiple layers (possibly containing periodic frequency selective surfaces) which themselves consist of materials which are isotropic on a micro-scale, then μ_e can only be different (greater than) unity if the composite contains (magnetic) particles whose relative permeability is greater than unity at zero frequency. For example, it is not possible to achieve $\mu_e > 1$ at zero frequency with metamaterials unless the metamaterials contain magnetic particles. Similar expressions have been obtained by others (e.g. [3]) for chiral materials.

In this article we will assume that the RAM composite contains no magnetic materials, i.e. that $\mu_e = 1$. This is because it is very difficult to construct materials which have appreciable values of μ_e at microwave frequencies and which have the necessary magnetic dispersion characteristics to provide optimality (given by the equality) in (1).

2 Performance Coefficients

The bandwidth integral (1) may be easily derived using forms by Fano [2] in his seminal 1950 paper. We will employ Fano's form (obtained using $\lambda = 2\pi c_0/\omega$, where c_0 is the speed of light in free space and ω is the angular frequency). In this form (for a non-magnetic RAM), we may define a merit coefficient $0 < B_p \leq 1$ where $B_p = 1$ under the above optimality definition for minimum reflection phase,

$$B_p = \frac{-1}{10log_{10}(e).\pi A_1^0} \int_0^\infty \frac{\Gamma_{dB}(\omega)}{\omega^2} d\omega \tag{2}$$

where $\Gamma_{dB}(\omega)$ is the reflection coefficient in dB as a function of the angular frequency and A_1^0 is Fano's coefficient. Using the zero frequency limit form for the input impedance of a multi-layer structure it may be shown that for an N-layer composite containing FSS and N isotropic layers,

$$A_1^0 = \frac{2L_{tot}}{Z_0} \begin{cases} \cos\theta & \text{TE polarisation} \\ 1/\cos\theta & \text{TM polarisation} \end{cases} \tag{3}$$

where Z_0 is the free space impedance and L_{tot} is the shunt inductance of the equivalent circuit of the input impedance of the RAM, defined by

$$L_{tot} \leq \frac{Z_0}{c_0} \sum_{l=1}^{N} d_l \hat{f}_l \tag{4}$$

where d_l is the thickness of each layer and $\hat{f}_l$ is a polarisation dependent modification factor,

$$\hat{f}_l = \begin{cases} 1 & \text{TE case} \\ 1 - \sin^2\theta/\epsilon_l & \text{TM case 0} \\ 1 & \text{TM case 1} \end{cases} \tag{5}$$

where ϵ_l is the (real) relative permittivity of layer l at zero frequency. TM case 0 represents the TM polarisation with layer l a lossless dielectric material (zero conductivity) at zero frequency. TM case 1 is where there is non-zero conductivity at zero frequency. There is equality in (4) if the RAM contains no 'slotted' FSS or (depending on the geometry) perfectly conducting 'vias' between interfaces. We define a 'slotted' FSS as one whose equivalent surface impedance features a shunt inductance, with a geometry which contains conducting current paths across unit cell boundaries. The absence of additional shunt inductance is what determines whether the presence of vias, too, is sub-optimal.

In addition to B_p-optimality (minimum reflection phase), it is frequently required that the reflection coefficient be close to rectangular at normal incidence. Assuming such a rectangular distribution with $B_p = 1$,

$$\pi A_1^0 = 0.1\, S_{dB} \log_e(10) \left(\frac{1}{\omega_l} - \frac{1}{\omega_u} \right) \tag{6}$$

where S_{dB} is the absorption depth in dB, defined positive, ω_l is the lower angular frequency band edge and ω_u is the upper frequency band edge. Suppose we define some further parameters,

$$M_\alpha = \frac{\lambda_0^{(max)}}{d_T} \tag{7}$$

representing the relative thickness of the RAM compared to the largest wavelength over the band, $\lambda_0^{(max)} = 2\pi c_0/\omega_l$ and

$$M_\beta = \frac{\omega_u}{\omega_l} \tag{8}$$

representing the bandwidth. For a general non-rectangular distribution, we may therefore define the normal incidence rectangular merit coefficient,

$$M_p = K \max_{M_\alpha^{(r)}, M_\beta^{(r)}} \left\{ S_{dB}^{(r)} M_\alpha^{(r)} \left(1 - \frac{1}{M_\beta^{(r)}} \right) \right\} \tag{9}$$

where K is the dimensionless constant,

$$K = \frac{0.1\, \log_e(10)}{4\pi^2} \approx 5.83 \times 10^{-3} \tag{10}$$

and $M_\alpha^{(r)}$, $M_\beta^{(r)}$ and $S_{dB}^{(r)}$ represent the values of the non-superfixed quantities that define a rectangle that just fits within the non-rectangular distribution, i.e. where $-\Gamma_{dB}(\omega) \leq S_{dB}^{(r)}$ for all frequencies $\omega_l \leq \omega \leq \omega_u$. The quantities are chosen so as to maximise M_p. The total composite thickness, $d_T = \sum_{l=1}^{N} d_l$. In general $0 < M_p \leq 1$ with equality only if $B_p = 1$.

It is interesting to assess RAM performance using this merit figure. For example

Munk (see section 9.9.1 of [4]) claims an FSS RAM with a rectangular distribution with merit figures $M_\alpha = 8.0$, $M_\beta = 10.0$ and $S^{(r)}_{dB} = 25.0$. This leads to a figure for $M_p \approx 1.05$; i.e. an FSS RAM which is (given that the accuracy of the figures is not specified) close to optimal.

3 Establishing $B_p = 1$ at normal incidence

The use of equivalent circuits to represent the input impedance of a general RAM is very important in optimal RAM design. In general, if the input impedance Z_{in} is well approximated (over some useful frequency range) by a finite order model then, taken as a function of the Laplace transform variable $p = j\omega$,

$$Z_{in}(p) \approx \frac{P(p)}{Q(p)} \tag{11}$$

where $P(p)$ and $Q(p)$ are real valued polynomial functions of p. Given our assumptions of linearity and passivity, circuit theory gives a number of constraints on these polynomials (e.g. [5]). The finite order rational function approximation implies that the frequency characteristic of any realiseable RAM may be exactly represented by an equivalent circuit featuring a finite number of resistors, capacitors and inductors. Any realiseable RAM may be described by an input impedance whose equivalent circuit is represented by an inductance L_{tot} in parallel with some other realiseable impedance function, so $Z_{in}(\omega) \to 0$ as $\omega \to 0$.

If, in addition, the reflection coefficient is minimum phase (which is required for $B_p = 1$) then at normal incidence a necessary and sufficient condition is that the polynomial, $P(p) - Z_0 Q(p)$ must be Hurwitz stable. There are several tests for Hurwitz stability in control theory which may readily be applied, e.g. the theorems of Routh-Hurwitz or (often more useful) of Strelitz [6]. If an equivalent circuit is available with a specified topology, Hurwitz stability will exist (if at all) only for certain parameter ranges. For low order equivalent circuits and for certain special classes (e.g. ladder networks of certain forms), analytical expressions for these parameter ranges may be provided.

A very useful diagnostic is the Nyquist diagram. It can be shown that $B_p = 1$ if and only if the normalised input impedance, $Z_{in}(\omega)/Z_0$, taken as a function of ω rather than p, does not loop the point $(1, 0)$ in the complex plane (in the sense, more precisely, that the total loop count should be zero).

3.1 Single resonance absorption

It is readily possible to construct a thin narrowband absorber whose reflection coefficient is described by a single resonance. Examples include lossy patch or dipole FSS arrays on a thin lossy substrate (backed by a perfect conductor). A general equivalent circuit representation for a capacitive FSS on a thin substrate is illustrated in the figure below. The rational function representation is bi-quadratic, where the inductance L is dominated by the perfect conductor terminated substrate acting as a shorted transmission line. For a thin material $L \approx d_T Z_0 / c_0$, and the C and R terms depend on the FSS and the surrounding lossy medium.

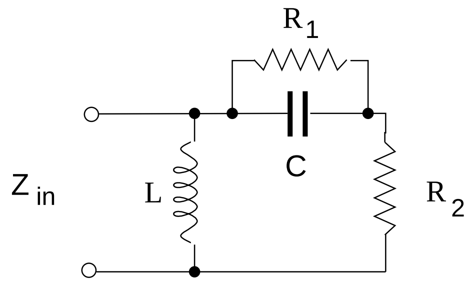

Figure 1: *Equivalent circuit for a thin capacitive FSS RAM.*

It is readily shown that $B_p = 1$ if and only if,

$$\begin{aligned} R_2 &\leq Z_0 \\ R_1 + R_2 &\leq Z_0(1 + C R_1 R_2 / L) \end{aligned} \quad (12)$$

If the FSS elements introduce a small but significant inductance, the above equivalent circuit is not valid and $B_p = 1$ is often not achieved. However, the design of rectangular shaped characteristics with $B_p \approx 1$ and large values of M_p *requires* the FSS to have inductive components.

3.2 Shaped (rectangular) characteristics

Near optimal RAM with a near rectangular distribution may in principal be obtained either with a single layer and single FSS or with multiple layers and multiple FSS. The circuit analogue RAM of the kind described by Munk [4] fall into the latter catagory. For design of such structures it is useful to establish classes of equivalent circuits which (a) can be realised by an FSS composite, (b) have the property that $B_p = 1$ and (c) allow control of the shape of the characteristic while maintaining $B_p = 1$. Once established, full wave numerical optimisation techniques can be used in a robust manner.

Exact analysis of rectangular distribution filters with the property of minimum reflection phase is restricted to a few classes of ladder networks. As far as we are aware, Fano [2] was first to describe a Tchebysheff ladder network for which $B_p = 1$ and for which the input of the network features a shunt inductor, necessary for modelling any RAM. It is possible, using more recent filter theory (e.g. as described by Rhodes [7]), to derive explicit circuit parameters for more general ladder networks meeting these requirements. For example, beginning with an original low-pass Tchebysheff ladder network of inductors and inverters, explicit component values have been derived for the band-pass equivalent circuit illustrated below. This circuit is able to describe a single FSS composite, provided the RAM has a thickness no greater than $\sim \lambda_0/3$ at the smallest wavelength of operation.

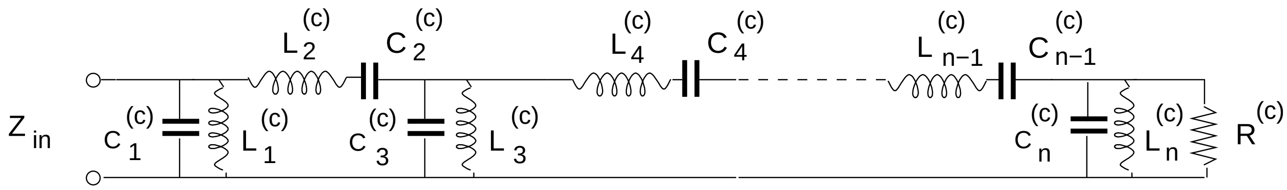

Figure 2: *Equivalent circuit for a rectangular distribution single FSS RAM.*

The component $L_1^{(c)}$ represents the shunt inductance of the substrate, $C_1^{(c)}$ represents the combined effect of the shunt capacitance of the substrate and the FSS, while the remaining components should represent the FSS. The coefficient $M_p \to 1$ as the filter order $n \to \infty$. Figure 3 illustrates a full wave prediction using QDAS [8] based on a single FSS RAM implementation of a fourth order equivalent circuit of this kind. This features a reactively loaded square patch array (using discrete or sub-scale reactive elements) with a RAM thickness $d_T = 15.0$ mm. The centre frequency is ~ 1.1 GHz, $B_p \approx 1.0$, $M_p \approx 0.80$, $M_\alpha \approx 27$, $M_\beta \approx 1.9$ and $S_{dB}^{(r)} \approx 10.7$.

3.3 Non-canonical methods

We define a 'non-canonical' method as one employing a structure whose equivalent circuit does not fall into a class which permits an exact analysis. To our knowledge, no analytic results are available for the design of $B_p = 1$ electrically thick FSS composite structures. "Circuit analogue Jaumann" absorbers fall into this catagory, but are useful since they may be operated over a very wide bandwidth. Although there is no proof on the maximum achievable value for M_p, numerical optimisation can be very effective if each FSS is represented by a surface impedance of relatively simple form. Figure 4 shows an example of the reflection coefficient of a 5-layer circuit analogue RAM. This assumes that each FSS is represented by a parallel combination of two series LCR circuits embedded in free space. No attempt is made to show how such FSS should be constructed in this example. The total composite thickness $d_T = 15.0$ mm. Here we obtain merit coefficients $M_p \approx 0.75$, $M_\alpha \approx 6.1$, $M_\beta \approx 14$ and $S_{dB}^{(r)} \approx 23$.

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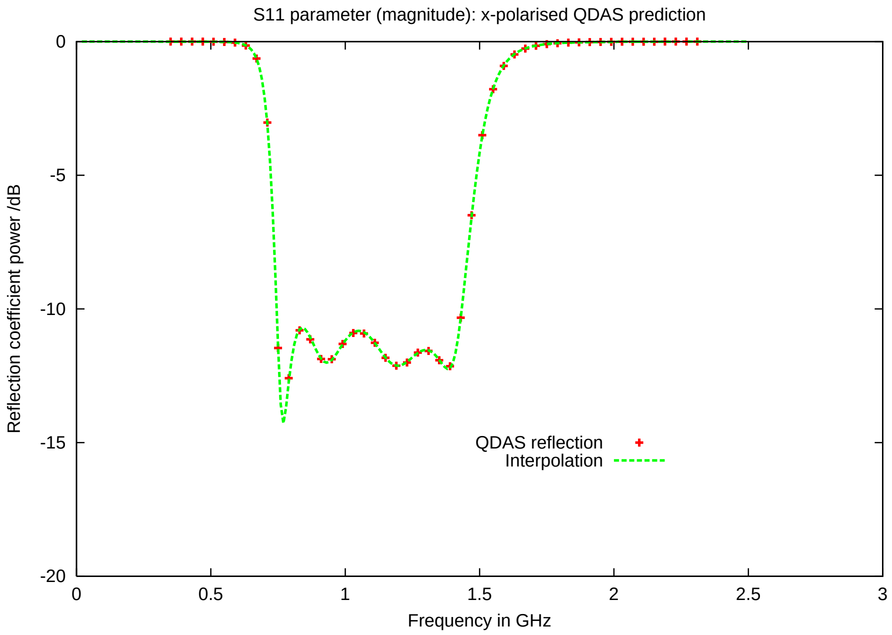

Figure 3: *Predicted reflection coefficient of an electrically thin FSS RAM.*

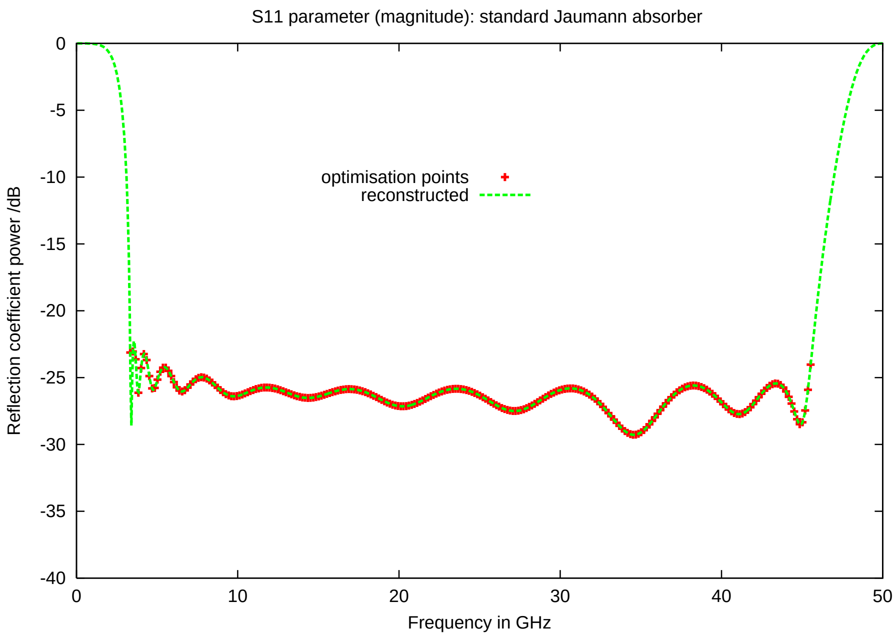

Figure 4: *Predicted reflection coefficient of a multi-layer wideband circuit analogue FSS RAM.*

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